Condensation in a zero range process on weighted scale-free networks

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We study the condensation phenomenon in a zero range process on weighted scale-free networks in order to show how the weighted transport influences the particle condensation. Instead of the approach of grand canonical ensemble which is generally used in a zero range process, we introduce an alternate approach of the mean-field equations to study the dynamics of particle transport. We find that the condensation on the scale-free network is easier to occur in the case of weighted transport than in the case of weight-free networks. In the weighted transport, especially, a dynamical condensation is even possible for the case of no interaction among particles, which is impossible in the case of weight-free networks.

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I. INTRODUCTION

Condensation, a concept originally introduced by Bose and Einstein to explain the particle condensation in momentum space, is an intriguing phenomenon observed in real space, such as jamming in traffic communication [1,2], bunching of buses [3], and many mass transport models [4]. In condensation, a finite fraction of particles may be condensed onto a single site. Nowadays, such phenomenon is found in the structure of complex networks, which characterize many natural and manmade systems, such as the Internet, airline transport system, power grid infrastructures, and the world-wide web [5,6]. For example, Bianconi and Barabasi mapped the fitness model to a Bose gas by assigning an energy to each node and found that the fittest node can attract a finite fraction of all links [7,8]. Besides the static condensation of the links of complex networks, the dynamical condensation on scale-free (SF) networks is found recently that the particles may completely condense on the hub in the transport of zero range process (ZRP), where the interaction only occurs when the particles are stay at the same node [9,10].

The previous studies of ZRP are mainly focused on regular lattices [4,11,12]. It is found that, in the steady state of the one-dimensional lattice, a finite fraction of the total mass will condense onto a single site when the global mass density is increased beyond a critical value. However, most of the realistic networks are not regular but SF [5,6]. Comparing to the regular lattice, the SF network is heterogeneous with power-law degree distribution $P(k) \sim k^{-\gamma}$ and can be characterized by the Barabasi-Albert (BA) model with $\gamma=3$ [13] and its related modified models [14–18]. It is pointed out that the structure inhomogeneity is an important factor in understanding many dynamical processes in SF networks [19–21], such as rumor propagation, virus spreading, and searching information, etc. For studying the influence of structure inhomogeneity on ZRP, Noh *et al.* considered ZRP in the BA model and found that the inhomogeneous structure makes particles to condense on the hub when the jumping rate δ is smaller than the critical point δ_c [9,10]. For a specific particle, its behavior can be considered as a random walk; and for a large number of particles, their behaviors are a kind of diffusion processes. As the diffusion makes the number of particles on a node fluctuate, the usually used approach to deal with the transport statistical properties in ZRP is the grand canonical ensemble, which can easily give the mean particles on each node. Recently, Ref. [4] uses the approach of canonical ensemble to explore the condensed phase and analyze the mass distribution. Here we introduce an alternate approach to deal with the ZRP, i.e., the mean-field equations, and find that our approach will give the same results as that given by the approach of grand canonical ensemble [9,10], when it is used in the same situations. Moreover, we use this approach to study the case of weighted transport and find that the added weight may push forward the critical value δ_c to a larger value δ_c', and δ_c' may be even over unity!

Let us call the network with the same weight to transfer particles on each link as a *weight-free* network. Besides a weight-free network, there is also a weighted network where each link has different weight to transfer particles, which yields a more realistic description of communication in real networks. Reference [9,10] investigated how the network structure influences particle dynamics. Our motivation here is to study how the weight of link influences the particle dynamics. In the weighted network, each link or node is associated with a weight w_i. The weight may represent the intimacy between individuals in social networks, or the bandwidths of routers and optical cables in the Internet. The strength of a node, s_i, is defined as the sum of all the weights of links of the node. It is found that the strength strongly relies on its degree with $s(k) \sim k^{\alpha}$ [22–24], where α is different constants for different networks. For example, α is unity for the science cooperation network where the weight of a link between two scientists is given roughly by the frequency of their collaboration and 1.5 for the world-wide airport network where the weight is taken as the total number of passengers of the direct flights between two connected cities. The previous results on condensation focus on the case of $\alpha=0$ [9,10]. In this paper, we consider the particle transport on the weighted network, i.e., the case of $\alpha>0$. Our results show that the condensation is easier to occur in the case of $\alpha>0$ than that of $\alpha=0$. Especially since the condensation may occur even when the interaction among particles does not exist, which is impossible in the case of $\alpha=0$.

The paper is organized as follows. In Sec. II, we briefly review the approach of grand canonical ensemble and its results of condensation on SF networks with weight-free transport. Then in Sec. III, we give the dynamical mean-field equations for the weighted transport on SF networks. Its limit will give the results of weight-free transport. In Sec. IV, we make numerical simulations to confirm the predictions given in Sec. III. Finally, the conclusions are given in Sec. V.

II. THE APPROACH OF GRAND CANONICAL ENSEMBLE FOR THE CASE OF WEIGHT-FREE TRANSPORT

The approach of grand canonical ensemble was first applied in the ZRP of a one-dimensional system [4,11,12,25] and recently used in the SF networks [9,10]. Here we briefly review the results obtained in [9,10]. Suppose N particles are randomly put on a network of L nodes and each node i can be occupied by any integral number of particles $n_i=0,1,2,\ldots,N$. Because of the interaction among the particles at the same node, some of the particles will jump out of the node and hop to other nodes, making the particle redistribution among $n_1,n_2,\ldots,n_L$. Hence a microscopic configuration is represented by $\mathbf{n}=n_1,n_2,\ldots,n_L$. The particles at node i with $n_i>0$ may jump out with the jumping rates $p(n_i)$ and hop from node i to one of its neighbors j along the link with the hopping probability $T_{j\leftarrow i}$. In the SF networks with weight-free, $T_{j\leftarrow i}$ is taken as $1/k_i$ if i and j are linked and 0 otherwise, i.e., a particle jumping out of the node i is allowed to hop to one of its neighboring nodes selected randomly, and the jumping rate is taken as $p(n_i)=n_i^{\delta}$. $\delta=0$ means that only one of n_i particles will jump out per each time, indicating that the particles are attracting each other. In the case of $\delta=1$, all the n_i particles will jump out, implying they are moving independently and the system reduces to a noninteracting system of N random walkers. $\delta>1$ means there is a repulsive interaction among particles of one node, while $\delta<1$ means the attractive interaction among particles. With enough time of evolution, the system will reach a stationary state. The mean occupation number in the stationary state is

$$m_i(z) = x \left.\frac{\partial \ln F_{\delta}(x)}{\partial x}\right|_{x=zk_i}, \tag{1}$$

where z denotes the fugacity and

$$F_{\delta}(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n!)^{\delta}}. \tag{2}$$

It is found that the complete condensation occurs at $\delta=0$, where the whole fraction of particles is concentrated at the hub [9,10], while there is no condensation at $\delta=1$. For the case of $\delta>0$, there is a critical

$$\delta_c = 1/(\gamma-1), \tag{3}$$

where the condensation will occur for $\delta\leqslant\delta_c$ and does not occur when $\delta>\delta_c$. There is a crossover degree k_c for $0<\delta\leqslant\delta_c$, which is defined as the degree with the average occupation number $m_i=1$. m_i will be greater than 1 for the nodes with $k_i>k_c$ and smaller than 1 for the nodes with $k_i<k_c$. There will be a condensation when there exists a finite $k_c>0$ and no condensation otherwise. For $\delta\leqslant\delta_c$, k_c can be expressed as

$$k_c \sim \begin{cases} (\ln k_{max})^{\delta_c} & \text{for } \delta=\delta_c \\ (k_{max})^{1-\delta/\delta_c} & \text{for } \delta<\delta_c, \end{cases} \tag{4}$$

where the nodes with $k>k_c$ will be the core of condensation, i.e., most of particles will hop to those nodes with $k>k_c$.

Furthermore, there is a scaling

$$m_i = G_{\delta}(k_i/k_c), \tag{5}$$

where the scaling functions behaves as $G_{\delta}(y\ll 1)\sim y$ and $G_{\delta}(y)(y\gg 1)\sim y^{1/\delta}$.

III. DYNAMICAL MEAN-FIELD EQUATIONS FOR THE CASE OF WEIGHTED TRANSPORT

According to the strength distribution $s(k)\sim k^{\alpha}$ [22,23], the strength is a nonlinear function of k and the nodes with the same k have the same strength. The strength will be different for the k_i neighbors of node i. When a jumping out particle hops to one of its neighbors, it will not choose all the links of the node on equal footing but with different rates to different neighbors. Therefore, we here assume the hopping rate $T_{j\leftarrow i}\sim k_j^{\alpha}$. Making renormalization, we have

$$T_{j\leftarrow i} = \frac{k_j^{\alpha}}{\sum_{j'\in B_i} k_{j'}^{\alpha}}, \tag{6}$$

where B_i denotes the set of neighbors of node i and Eq. (6) will return to the case of weight-free networks when $\alpha=0$.

We still suppose here that the jumping rates $p(n_i)$ satisfy $p(n_i)=n_i^{\delta}$ and restrict δ to $0\leqslant\delta\leqslant 1$. With the evolution, n_i will change with time and may be different for different nodes with the same k. For using the mean-field approach, we transfer the description of n_i for each node to the description of the mean occupation number $m_k(t)$ for the nodes with the same k, i.e., $m_k(t)$ is the average of all the n_i of the nodes with degree k. Hence, $m_k(t)$ is no longer necessary to be an integer. Correspondingly, we transfer the jumping rate $p(n_i)$ to $p(m_k)=m_k^{\delta}$. After this transformation, let us see how many particles of a node can jump out at each time step. When $m_k\leqslant 1$, $m_k^{\delta}\geqslant m_k$ and the jumping rate is, in fact, m_k but not m_k^{δ} as we do not have so many average particles m_k^{δ} at those nodes with degree k. When $m_k>1$, $m_k^{\delta}<m_k$ and hence the jumping rate is m_k^{δ}. Except the aspect of jumping out, at the same time, a node accepts particles from its neighbors. The incoming particles can be classified into two parts: One from the node with $m_k<1$ and the other from the nodes with $m_k\geqslant 1$. The incoming particles from one neighboring node with degree k' is $P(k'|k)m_{k'}(t)k^{\alpha}/\Sigma_{j\in B_{k'}}k_j^{\alpha}$ when $m_{k'}(t)<1$ and $P(k'|k)m_{k'}^{\delta}(t)k^{\alpha}/\Sigma_{j\in B_{k'}}k_j^{\alpha}$ when $m_{k'}(t)\geqslant 1$, where $P(k'|k)$ is the conditional probability for a link which represents the possibility for a node with degree k to connect a node with degree k'. We use k_0 and k_{max} to represent the minimum and maximum degree of the network, respectively, and use $k_c(t)$ to denote the degree for $m_k(t)=1$. In the BA model, k_0 is a constant and $k_{max}\sim L^{\beta}$ with $\beta=1/(\gamma-1)$. By all these quantities, we have the mean field equations for the evolution of $m_k(t)$

$$\frac{\partial m_k(t)}{\partial t}=-m_k(t)+k\left(\sum_{k_0}^{k_c}P(k'|k)m_{k'}(t)\frac{k^{\alpha}}{\sum_{j\in B_{k'}}k_j^{\alpha}}+\sum_{k_c}^{k_{max}}P(k'|k)m_{k'}^{\delta}(t)\frac{k^{\alpha}}{\sum_{j\in B_{k'}}k_j^{\alpha}}\right),\quad m_k(t)<1,$$

$$\frac{\partial m_k(t)}{\partial t}=-m_k^{\delta}(t)+k\left(\sum_{k_0}^{k_c}P(k'|k)m_{k'}(t)\frac{k^{\alpha}}{\sum_{j\in B_{k'}}k_j^{\alpha}}+\sum_{k_c}^{k_{max}}P(k'|k)m_{k'}^{\delta}(t)\frac{k^{\alpha}}{\sum_{j\in B_{k'}}k_j^{\alpha}}\right),\quad m_k(t)\geq 1,\quad (7)$$

where $\Sigma_{j\in B_{k'}}k_j^{\alpha}$ denotes the sum to all the neighbors of the node with degree k'. Obviously, $m_k(t)$ depends on $P(k'|k)$. For getting the detailed form of $P(k'|k)$, here we consider the BA model as the underlying network. As the BA model is nonassortative mixing, its conditional probability satisfies $P(k'|k)=k'P(k')/\langle k\rangle$ [26,27]. Borrowing the definition of the average degree of the nearest neighbors of a node with degree k, i.e., $\bar{k}_{nn}(k)=\Sigma_{k'}k'P(k'|k)$, we have the average

$$\bar{k}_{nn}^{\alpha}(k)=\int_{k_0}^{k_{max}}P(k'|k)k'^{\alpha}dk'=\langle k^{\alpha+1}\rangle/\langle k\rangle.\quad (8)$$

It is easy to see that $\bar{k}_{nn}^{\alpha}$ does not depend on the degree k. Hence, we have $\Sigma_{j\in B_{k'}}k_j^{\alpha}=k'\bar{k}_{nn}^{\alpha}$ and Eq. (7) becomes

$$\frac{\partial m_k(t)}{\partial t}=-m_k(t)+k^{\alpha+1}A(t),\quad m_k(t)<1$$

$$\frac{\partial m_k(t)}{\partial t}=-m_k^{\delta}(t)+k^{\alpha+1}A(t),\quad m_k(t)\geq 1\quad (9)$$

where $A(t)=[\Sigma_{k_0}^{k_c}P(k')m_{k'}(t)+\Sigma_{k_c}^{k_{max}}P(k')m_{k'}^{\delta}(t)]/(\langle k\rangle\bar{k}_{nn}^{\alpha})$.

Equation (9) is the evolution equation of particles and will reach the stationary state when it evolves enough time. In the stationary state, we may solve Eq. (9) by $\partial m_k(t)/\partial t=0$ to get the stabilized m_k. By doing this we have

$$m_k=k^{\alpha+1}A,\quad m_k<1$$

$$m_k=(k^{\alpha+1}A)^{1/\delta},\quad m_k\geq 1\quad (10)$$

where $A=[\Sigma_{k_0}^{k_c}P(k')m_{k'}+\Sigma_{k_c}^{k_{max}}P(k')m_{k'}^{\delta}]/(\langle k\rangle\bar{k}_{nn}^{\alpha})$ does not depend on the time t. We have $A=k_c^{-(\alpha+1)}$ from the condition $m_k=1$ for $k=k_c$. Instituting A into Eq. (10) we have

$$m_k=(k/k_c)^{\alpha+1},\quad m_k<1$$

$$m_k=(k/k_c)^{(\alpha+1)/\delta},\quad m_k\geq 1.\quad (11)$$

Obviously, Eq. (11) will go back to Eq. (5) when $\alpha=0$.

Recall that in the case of a one-dimensional lattice, one necessary condition for the condensation is that its density $\rho=N/L$ should be larger than a critical value $\rho_c>0$. While in the BA model of weight-free networks, there is no ρ_c for ρ. The condensation occurs at any finite value of the particle density, i.e., $\rho_c=0$, and the particles can completely condense at the hub or a few high-degree nodes [9,10]. Here the limit case of $\alpha=0$ is the BA model of weight-free networks, hence we expect $\rho_c=0$ for $\alpha>0$, which will be demonstrated later. For confirming it, in the case of weighted transport we set the particle density $\rho<1$. In the limit of $L,N\to\infty$ we have $m_k<1$ for all nodes if there is no condensation; otherwise, part of m_k will become greater than unity. Therefore, the k_c for $m_k=1$ is nothing but the crossover degree. The number of condensed particles is

$$N_{con}\sim N\int_{k_c}^{k_{max}}(k/k_c)^{(\alpha+1)/\delta}P(k)dk.\quad (12)$$

In the condensed state, the nodes with $k_c\leq k\leq k_{max}$ have the capacity to take an infinite number of particles. The nondivergence of the integration in Eq. (12) gives

$$\delta_c'=\frac{\alpha+1}{\gamma-1}.\quad (13)$$

The condensation will occur for the case of $\delta<\delta_c'$. From Eq. (13) one can see that in the case of $\alpha=0$, $\delta_c'=1/(\gamma-1)$ returns to the case of weight-free networks of Eq. (3). By $N_{con}/N\sim O(1)$, we have

$$k_c^{(\alpha+1)/\delta}\sim\int_{k_c}^{k_{max}}k^{(\alpha+1)/\delta}k^{-\gamma}dk,\quad (14)$$

and further we have

$$k_c\sim\begin{cases}(\ln k_{\max})^{\delta_c} & \text{for } \delta=\delta_c'\\ (k_{max})^{1-\delta/\delta_c'} & \text{for } \delta<\delta_c'.\end{cases}\quad (15)$$

Obviously, it has the same form with Eq. (4) except δ_c is replaced by δ'. From Eq. (15) one can see that k_c does not depend on the particle density ρ, indicating that the condensation can occur at any ρ. Therefore, we also have $\rho_c=0$ for the weighted transport.

For the situation of $\delta<\delta_c'$, from Eq. (11) the particles accumulated at the hub are

$$m_{hub}\sim(k_{max}/k_c)^{(\alpha+1)/\delta}\sim(k_{max}^{\delta/\delta_c'})^{(\alpha+1)/\delta}\sim L,\quad (16)$$

indicating that $m_{\rm hub}$ scales linearly with L and hence there is condensation at the hub. And for the situation of $\delta>\delta_c'$, the particles at the hub are

$$m_{hub} \sim k_{max}^{(\alpha+1)/\delta} \sim L^{\delta_c'/\delta}, \tag{17}$$

indicating m_{hub} scales sublinearly with L and hence there is no condensation. When there is condensation, from Eq. (11) we have

$$m_k \sim k^{\eta} \tag{18}$$

with $\eta=(\alpha+1)/\delta$ for $\delta<\delta_c'$ and $m_k>1$. The scaling η will decrease to $(\alpha+1)/\delta_c'=\gamma-1$ when δ increase to δ_c', which gives a critical $\eta_c=2$ for $\gamma=3$. Therefore, we can judge the occurrence of condensation by checking the scaling η for $m_k>1$, i.e., there is condensation if $\eta>\gamma-1$ and no condensation otherwise.

Moreover, from Eq. (13) we can see that, for the BA model with $\gamma=3$, it is possible to have $\delta_c'>1$ for $\alpha>1$. In this case the condensation will occur for the whole range of $0\leqslant\delta\leqslant 1$, which is impossible in the case of weight-free networks. For example, taking $\delta=1$ for the case of weight-free networks, all the particles at node i will jump out of the node per each time, resulting in a uniform distribution of particles and hence no condensation. However, in the case of weighted transport, the situation is totally changed. During the evolution, all the particles will jump out at each time and hop with the favorite to one of the neighbors with the largest degree. Gradually, most of the particles will move to the larger and larger nodes, and finally to around the hub. After that, all the particles at the hub will jump out per each time when $\delta=1$, but at the same time, most of the particles of the hub's neighboring nodes will hop to the hub, resulting in a dynamical equilibrium. Namely, $\delta=1$ for $\delta_c'>1$ is a kind of completely *exchange* condensation, which is different from the case of $\delta<1$, where most of the previous particles at the hub will still stay at the hub. This kind of complete exchange condensation also exists for the case of $1<\delta<\delta_c'$, where the jumping rate $p(m_k)=m_k$ is the same with that of $\delta=1$.

IV. NUMERICAL SIMULATIONS

In numerical simulations, we first construct a BA growing network with the size $L=2000$, the average degree $\langle k\rangle=6$, and the degree distribution $P(k)\sim k^{-3}$ according to the algorithm given in Refs. [13,18]. Then we put $N=1000$ particles at the L nodes randomly, i.e., the particle density $\rho=0.5$. Hence $m_k(t=0)$ is uniform for different k, see the horizontal line for the case of $t=0$ in Fig. 1. At each time step, we let the N particles evolve according to the jumping rate $p(n_i)$ and the hopping rate $T_{j\leftarrow i}$ in Eq. (6). Namely, we choose $\min(n_i,n_i^{\delta})$ particles from the n_i particles of node i and let them hop to the neighbors of node i according to Eq. (6), where $\min(n_i,n_i^{\delta})$ means taking the smaller one from n_i and n_i^{δ}. By this way, we find that the numerical simulations completely confirm the evolution Eq. (9) and the stationary distribution Eq. (11). For example, from Eq. (9) we have $m_k(t=1)=A(0)k^{\alpha+1}\sim k^{\alpha+1}$, indicating a power-law relation between m_k and k for all the k at $t=1$. Our numerical simulation has confirmed it; see the straight line for the case of $t=1$ in Fig. 1 where the parameters are taken as $\alpha=0.4$ and $\delta=0.2<\delta_c'=0.7$. This algebraic relation will be kept until the

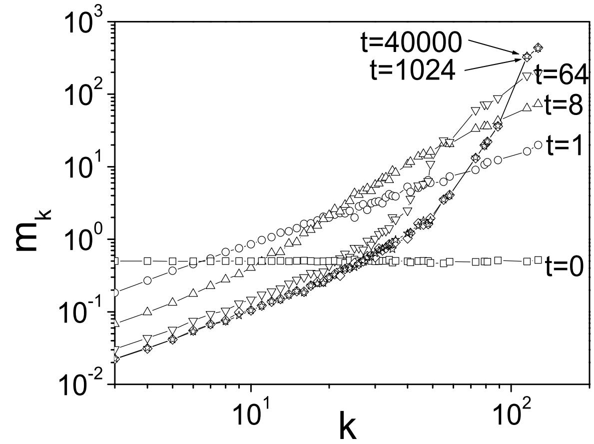

FIG. 1. Evolution of particle distribution in weighted transport for an arbitrary $\alpha=0.4$ and $\delta=0.2<\delta_c=0.7$.

emergence of some $m_k(t)>1$. After that, we know from Eq. (9) that the nodes with $m_k(t)<1$ for small k will keep the algebraic relation $m_k(t)\sim A(t-1)k^{\alpha+1}$ but the nodes with $m_k(t)>1$ for large k will depend on both k and $m_k(t-1)$ because of the nonlinearity of the first term $m_k^{\delta}(t)$ in Eq. (9). The numerical confirmation is given by the curves for the cases of $t=8$ and 64 in Fig. 1. It is easy to see from these two curves that the part with smaller k has the same slope with that of the curve of $t=1$ and the part with larger k is complicated. After further evolution, the system will reach its stationary state. The lowest lines in Fig. 1 show the numerical

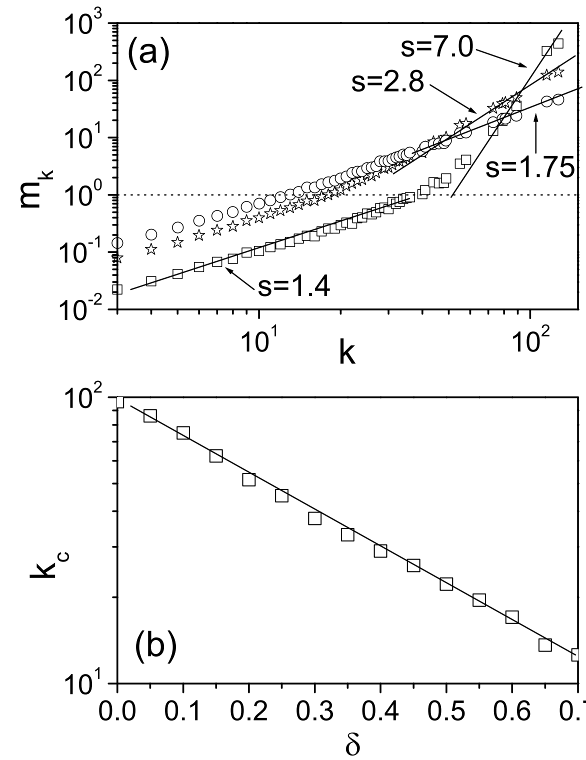

FIG. 2. (a) Stabilized m_k versus k for $\alpha=0.4$ and $\delta_c'=0.7$ where the lines with "squares," "stars," and "circles" denote $\delta=0.2$, 0.5, and 0.8, respectively. The drawn lines for slope $s=1.4$, 7.0, 2.8, and 1.75 are for reference. (b) Crossover degree k_c versus α.

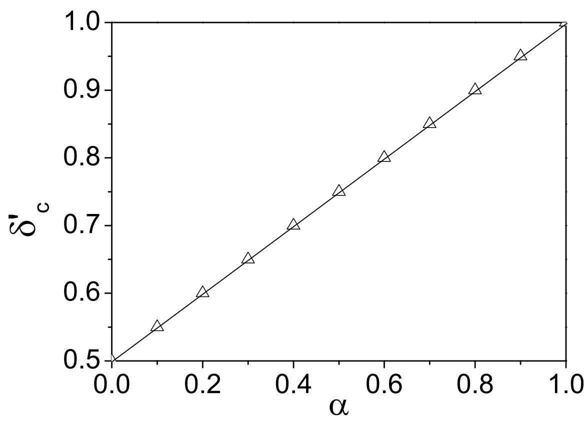

FIG. 3. The critical parameter δ_c' of condensation versus the weight parameter α.

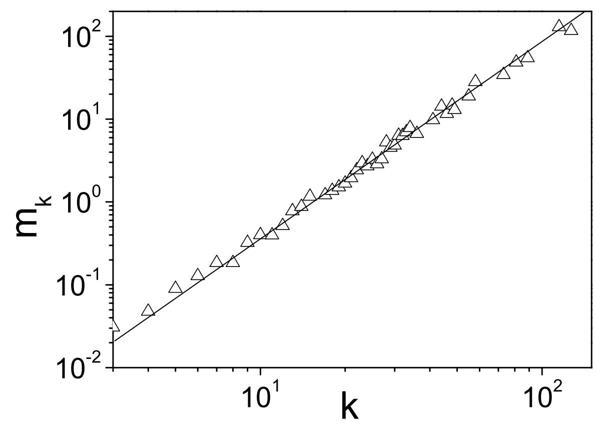

FIG. 4. The stabilized m_k versus k for $\alpha=1.4$ and $\delta=1<\delta_c'=1.2$.

results for $t=1024$ and 40 000, respectively. Obviously, the case of $t=1024$ is overlapped with the case of $t=40\,000$, indicating that the stationary state is reached.

In the stationary state, from Eq. (11) we know that the stabilized m_k will have two slopes and both depends on the weight scaling α, i.e., slope $\alpha+1$ for $m_k<1$ and slope $(\alpha+1)/\delta$ for $m_k>1$. The numerical simulations have completely confirmed this. Figure 2(a) shows the results for $\alpha=0.4$ and $\delta=0.2$, 0.5, and 0.8, respectively. By Eq. (13) we get $\delta_c'=0.7$, so the case of $\delta=0.2$ and 0.5 should have condensation and the case of $\delta=0.8>\delta_c'$ no condensation. This can be confirmed by checking the scaling η in Eq. (18). It is easy to see from Fig. 2(a) that the slopes are 7.0 and 2.8 for the cases of $\delta=0.2$ and 0.5, which are greater than $\eta_c=2$, and 1.75 for the case of $\delta=0.8$, which is less than 2. These values also confirm the prediction of Eq. (11) in which we have the slopes $\alpha+1=1.4$, and $(\alpha+1)/\delta=7$, 2.8, and 1.75 for $\delta=0.2,0.5$, and $\delta=0.8$, respectively, see the solid reference lines with different slopes s in Fig. 2(a). On the other hand, from Eq. (15) we have $\ln k_c \sim C\delta$. The crossover degree k_c can be measured from the intersection between the dotted line $m_k=1$ and the measured curves in Fig. 2(a). By this way we can get k_c for different δ. Figure 2(b) shows the result. It is easy to see that Fig. 2(b) is a straight line with negative slope, which qualitatively explains Eq. (15).

By the fact that $\eta>\eta_c$ means condensation, we may determine δ_c' numerically. For a given α, we increase δ gradually and check η for each δ. The critical δ_c' can be identified as the value that separates the $\eta>2$ and $\eta<2$. By this way, we obtain δ_c' for different α. Figure 3 shows the result. Obviously, it is a straight line with slope 0.5 and thus confirms Eq. (13). When α is over unity, however, we cannot obtain δ_c' numerically as δ is limited to unity for the jumping rate $p(n)$.

Furthermore, from Eq. (11) we can see that the two slopes for $m_k<1$ and $m_k>1$ will become the same when $\delta=1$. This phenomenon is also confirmed by our numerical simulation. Figure 4 shows the result where the measured slope 2.37 (the solid line) is consistent with the theoretical prediction $\alpha+1=(\alpha+1)/\delta=2.4$ and the fact that $2.37>2$ confirms the condensation for $\delta=1<\delta_c'$.

V. DISCUSSIONS AND CONCLUSIONS

The condensation in ZRP was usually discussed in mass transport and only little attention had been paid to the interacting dynamical systems on SF networks. The work of Noh *et al.* makes ZRP on SF networks become reality. As a complex network has very rich features, Noh's work is very important but only the beginning of ZRP on complex network. There are a plenty of factors in networks which may affect the condensation, such as the distribution, clustering coefficient, assortativity, and weight, etc. This paper investigates one of the factors. Our results show that the weight may significantly influence the condensation and make it possible for the condensation to occur even for $\delta=1$ when $\delta_c'>1$, which is impossible in the weight-free network. We will continue along this way.

In conclusions, we have discussed the dynamical condensation on the SF network with weighted transport. We have introduced a set of dynamical mean-field equations in ZRP to describe the evolution of a particle and find that the particle condensation depends on not only the jumping rate, but also the weight of transport. Our results show that the mean-field approach is completely equivalent to the grand canonical ensemble in the situation of $\alpha=0$ and the condensation is easier to occur in the weighted network than that in the network with weight-free. The critical value of condensation in weighted network is $\delta_c'=(\alpha+1)/(\gamma-1)$, which is larger than the value $\delta_c=1/(\gamma-1)$ of weight-free networks.

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